Recent advances in discounting: implications for forest economics

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Abstract

Discounting has played an important role in forestry economics because harvesting cycles are often much longer than project cycles for other investments. For high forests, for instance, most income is derived from thinning and felling up to 80 (sycamore and ash) or even 120 years (oak) after planting. Cost-benefit analysis of such long-term investments is enormously sensitive to the discount rate. Using conventional exponential discounting can generate recommendations that appear contrary to sustainability, if not commonsense. This paper reviews some recent advances in discounting theory, which suggests that social discount rates should decline over time, and applies this theory to three case studies to tease out the implications for forestry economics.

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1. Introduction: discounting and forest management

Forestry is a long-term investment. Harvesting cycles are often longer than 20 years, and can extend well beyond 100 years for certain timber products.[1] Whenever time horizons are long, the choice of discount rate for financial analysis becomes important.[2] For cycles of 100 years or more, the discount rate is paramount. A high discount rate — reflecting a high opportunity cost of investment — often means that investing now in order to reap a harvest decades into the future is simply not economically viable. Indeed, Pearce et al. (2002) state that '[h]igh discount rates in logging practice mean that there is little or no incentives to replant after one rotation, especially as there remains a substantial 'frontier' of unlogged forest to which the logging company can move'. Yet replanting can appear incredibly attractive if the discount rate is low. In this context, it would be extremely surprising if forest economists did not have a special interest in the conceptual basis for discounting and the selection of the discount rate. Indeed, concerns about discounting in forestry are hardly new. Thirty years ago, Klemperer (1976) noted that forest economists 'often fall into disfavour with those concerned about maintaining abundant future wood supplies. Most feared of all the economist's devilish instruments is the compound interest rate.'

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[1] See, for instance, financial analysis by De Graaf et al (2003) of the Terra Firme Forest in Amazonia, where a 100-year harvesting cycle is considered.

[2] For instance, the present value of timber revenue of £100 in 40 years time is almost £70 at a discount rate of 1%, but is only £10 at a discount rate of 6%.

The discount rate is not only important to the economic value of future timber products. Forestry provides humans with other direct use values (e.g. extraction of genetic material, tourism), indirect use values (e.g. protection of watersheds, support of other ecosystems and carbon storage), option values and non-use values (i.e. existence or passive use values). Some of these services, especially those derived from genetic material and carbon storage, provide benefits to generations living well over 100 years in the future. The discount rate is crucial in determining the net present value of such forestry benefits.[3]

Although the 'correct' discount rate is still debated, most of the current literature on optimal forest rotation has not questioned the assumption that a *constant* discount rate is appropriate.[4] Recently, economic theorists in several other areas have begun to reach the conclusion that constant discount rates are unjustified. For instance, work surveyed in Groom et al. (2005) suggests that the correct social discount rate should actually vary with time, beginning with the short-term rate of 3.5% and declining over the long run to 1%. The rationales for time-declining rates are varied, and often complex. But arguably the most important rationale derives from the fact that the future state of the economy,

[3] For instance, the present value of forestry benefits of £100 in 150 years time is approximately £20 at a discount rate of 1%, but is under 2p at a discount rate of 6%.

[4] A few exceptions that discuss discounting issues exist. For example, Lorrain-Smith (1982) argues that '[c]onventional discounting tables treat future events with decreasing significance until at some point they cease to have any consequence. When comparing alternative uses for resources, it should be appreciated that unless distant significant events are included in the calculations, their relevance may be ignored, and faulty decisions may result'.

and thus the appropriate discount rate, is uncertain. Under these conditions, it is relatively straightforward to show that the 'certainty equivalent' discount rate should decline with time. The effects of arguments such as this could be very significant for forestry economics.

This paper reviews the various challenges created by the issue of discounting in environmental economics (section 2), outlines a recent solution to those challenges involving the use of declining discount rates (section 3), traces the implications of this solution for forestry economics by examining three case studies (section 4), before concluding with some corresponding recommendations (section 5).

2. The issue of discounting in environmental economics

2.1 The social discount rate

In social cost benefit analysis, the social discount function, $D(t)$, is used to convert flows of future cost and benefits into their present equivalents.[5] If the net present value of the investment exceeds zero, the project is efficient. The social discount rate, $s(t)$, measures

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[5] This section focuses on social cost-benefit analysis for two reasons. First, it remains the case that a substantial proportion of forestry projects are conducted within the public sector, where the social discount rate is the applicable concept. Second, selecting the appropriate private discount rate is arguably a more straightforward conceptual exercise — it should reflect the opportunity cost of the firm, and general practice is to employ a weighted average cost of capital (Koller et al, 2005). Note, however, that the theory on uncertainty in section 3.4 below applies with equal force to the certainty-equivalent discount rate for the private sector.

the annual rate of decline in the discount function, $D(t)$. The two are connected by the equation:

$$D(t) = \exp\left[-\int_0^t s(\tau)d\tau\right]$$ (1)

A constant social discount rate implies that the discount function declines exponentially, $D(t) = \exp(-st)$.[6]

As practitioners know all too well, the choice of the social discount rate is often critical in determining whether projects pass social cost benefit analysis. The vigorous debates about the 'correct' discount rate were largely resolved by Lind (1982, p 89): the recommended approach is to `equate the social rate of discount with the social rate of time preference as determined by consumption rates of interest and estimated on the basis of the returns on market instruments that are available to investors'. Under this approach, the social discount rate, for a given utility function, can be expressed by the accounting relation:

$$s = \delta + \mu g$$ (2)

where δ is the utility discount rate (or the rate of pure time preference), μ is the elasticity of marginal utility and g is the rate of growth of consumption per capita. Even if the utility discount rate δ is zero, the social discount rate is positive when consumption growth, g, is positive and $\mu > 0$. Equation (2) shows that in general, the appropriate

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[6] The discrete time analogue of the discount function is the discount *factor*, given by: $D(t) = 1/(1+s)^t$.

social discount rate is *not* constant over time, but is a function of the expected future consumption path.

Note that the social discount rate given in equation (2) is not the same as market interest rates. Social discount factors are *shadow prices* of future consumption relative to consumption today. The market price for risk-free long-term interest rates is inappropriate as a conceptual basis for social discounting. There are several reasons for this. The most significant is that distortions in the economy, such as externalities, government taxation, imperfect information and market power (Drèze and Stern, 1990) imply that market prices do not reflect the true shadow price, or the true social opportunity cost of investing resources today.

This, of course, does not mean that market interest rates are entirely irrelevant. When public investment simply crowds out private investment, the opportunity cost of that investment is the market interest rate. However, public expenditure displaces private expenditure to a different extent depending upon the particular investment, and Lind (1982) recommends accounting for crowding-out effects 'by directly analyzing the magnitude of these effects and the converting them to their consumption equivalents through the use of a shadow price on capital.'[7]

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[7] This shadow pricing approach is not currently used in the UK (HM Treasury, 2003), reflecting a mix of practicability and the view that the real risk-free interest rate and the shadow discount rate are quite close in magnitude (Spackman, 1991; Pearce and Ulph 1999). See also Arrow (1995).

Another relevant feature in social cost-benefit analysis is uncertainty. Uncertainty is important on both sides of the analysis: (i) the investment itself has uncertain cash flows, and (ii) the social opportunity costs are uncertain. The second side is of greater interest here, and section 3.4 below examines the impact of uncertain social opportunity costs on the discount rate. The first side — uncertainty in investment cash flows — could simply be addressed by evaluating the possible scenarios, assigning probabilities, and then determining risk-adjusted certainty-equivalent cash flows.[8] The certainty-equivalent cash flows are just discounted at the appropriate rate, which reflects certainty-equivalent opportunity cost, or side (ii) of the analysis. Rather than bother determining certainty-equivalent cash flows, the private sector often takes a conceptual shortcut and applies a higher 'hurdle rate' (the required internal rate of return) to account for investment-specific risk. This no longer reflects guidance for public investment in the United Kingdom, where HM Treasury (2003) recommends against addressing project risk by adjusting the discount rate.

2.2 New discounting dilemmas

In recent years, debates about the correct foundation for the social discount rate have been replaced by controversy about intergenerational equity. Although this has been prompted by the challenges of climate change, biodiversity losses and nuclear waste

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[8] The appropriate risk adjustment, and whether it differs between the private and public sectors, has been debated at length in the literature, beginning with Hirshleifer (1964) and Arrow and Lind (1970). Grant and Quiggin (in press) provide a summary. However, the focus of this paper is on discounting, as reflecting opportunity costs, so we are less interested in the risk premia to be applied to specific investment risks.

management, issues of intergenerational equity are also extremely pertinent to long-cycle forestry projects.[9] It is now relatively widely agreed that conventional exponential discounting can yield results that appear to be contrary to intergenerational equity and sustainable development. For instance, the present value of £100 in 100 years time is £37 at a 1% discount rate, £5.2 at 3%, £2 at 4% and only 12p at 7%. Although this is obviously simply a reflection of the power of compound interest, this does create two problems. First, a small change in discount rate has a large impact on policy outcomes, meaning that arguments about the 'correct' number become more intense. Second, exponential discounting at even moderate discount rates implies that costs and benefits in the far future are almost irrelevant to decisions made today. While this might be entirely appropriate for individuals (who will no longer be alive), it is probably not a satisfactory basis for public policy.

These issues have spurred economists and policy makers to think more carefully about long-term intergenerational trade-offs. Some economists have advocated abandoning discounting altogether, proposing alternative methods to value the future.[10] Other

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[9] Many of these issues also involve potential irreversibilities, which might be thought to relate to discounting. In fact, provided the cost benefit analysis has appropriate addressed the costs (and benefits) of an irreversible change, including the loss of option value, there is no reason to employ a different discounting framework.

[10] For instance, the 'utility function approach' proposed by Schelling (1995) would avoid discount rates altogether by presenting policy makers with a menu of investments and a calculation of the utility increase in each world region (and time period) for each investment. Kopp and Portney (1999) and Page (2003) suggest using voting mechanisms. Page (1997) and Howarth (2003) propose positions based upon moral

economists conclude that although discounting (and cost-benefit analysis) is still very useful, it must be employed in a framework that guarantees intergenerational equity. A third view is that although conventional discounting is satisfactory for short-term decisions, it needs refinement before it can be legitimately used for long-term decisions. We investigate this third view in more depth in this paper. Before doing so, however, we address and dismiss a fourth position based on zero discounting.

2.3 Zero discounting?

Given the various controversies about discounting, it is tempting to suggest that we retain cost-benefit analysis, but simply not discount the cash flows in social cost-benefit analysis. But not discounting amounts to using a social discount rate of $s = 0\%$, which is dubious given the world has experienced positive consumption growth to date: $g > 0$ in equation (2). Very few, if any, economists could seriously argue that their central estimate of future consumption growth is 0%.[11]

In contrast, a credible argument for employing a zero *utility* discount rate ($\delta = 0$) can be advanced, based upon the ethical position that the weight placed upon a person's utility should not be reduced simply because they live in the future. Indeed, a string of eminent scholars have famously supported this position, including Ramsey (1928),

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duties which appear consistent with the view of Sen (1982) that the welfare economic framework is insufficiently robust to deal with questions of intergenerational equity

[11] That said, given data from the past few decades, achieving increases in human welfare and happiness appears to be a different matter altogether.

Pigou (1932), Harrod (1948) and Solow (1974), and even Koopmans (1965) expressed an `ethical preference for neutrality as between the welfare of different generations'.

However, not all philosophers and economists accept the presumption of impartiality. Arrow (1999), for instance, prefers the notion of 'agent-relative ethics' advanced by Scheffler (1982). This certainly provides a better *descriptive* account of human behaviour, and some would also find it a satisfactory *normative* position. Furthermore, there are four further arguments against impartiality that need to be addressed — the `no optimum' argument, the `excessive sacrifice' argument, the `risk of extinction' argument, and the `political acceptability' argument.

Hepburn (2006) reviews these arguments, and concludes that only the 'risk of extinction' argument provides a satisfactory conceptual basis for a positive utility discount rate. As Dasgupta and Heal (1979) note, `one might find it ethically feasible to discount future utilities as positive rates, not because one is myopic, but because there is a positive chance that future generations will not exist'. Nevertheless, one might speculate that the risk of exogenous social collapse would be rather small, probably (and hopefully) under 0.5% per annum (but c.f. Rees, 2003).

3. A recent resolution to the discounting dilemma?

The third view advanced above is that discounting remains a useful tool, but that it needs some adjustment before it is applicable to long-term projects. One of the most convincing adjustments is to employ a social discount rate that declines as time passes. Declining discount rates are not only appealing from the point of view of

intergenerational equity; they may even be *necessary* for achieving intergenerational efficiency. This section provides an overview of the main arguments for the case for declining discount rates.

3.1 Evidence on individual time preference

Evidence from experiments over the last couple of decades suggests that humans use a declining discount rate, in the form of a `hyperbolic discounting' function, in making intertemporal choices.[12] The shape of the discount function can be constructed by asking people to choose between a set of delayed rewards, such as money, durable goods, sweets or relief from noise.[13] The resulting discount functions suggest that humans employ a higher discount rate for consumption trade-offs in the present than for trade-offs in the future. While other interpretations, such as similarity relations (Rubinstein, 2003) and sub-additive discounting (Read 2001), are possible, the evidence for hyperbolic discounting is relatively strong.

Pearce et al. (2003) present the argument that if people's preferences count, and these behavioural results reveal underlying preferences, then declining discount rates ought to be integrated into social policy formulation. Pearce et al. recognise, however, that the assumptions in this chain of reasoning might be disputed. First, as hyperbolic discounting provides an explanation for procrastination, drug addiction, under-saving, and organisational failure, the argument that behaviour reflects preferences is

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[12] Interestingly, evidence suggests that some animals do likewise. Green and Myerson (1996) and Mazur (1987) provide summaries of evidence on the behaviour of birds.

[13] See, for instance, Thaler (1981), Cropper et al. (1994), Kirby (1997), Harris and Laibson (2001) and the reviews by Frederick et al (2002) and Ainslie (1992).

weakened. Second, Pearce et al. point out that Hume would resist concluding that the government *should* discount the future hyperbolically because individual citizens *do*. The recent literature on `optimal paternalism' suggests, amongst other things, that governments may be justified in intervening not only to correct externalities, but also to correct `internalities' — behaviour that is damaging to the actor.[14] Whether or not one supports a paternalistic role for government, one might question the wisdom of adopting a schedule of discount rates that explains procrastination, addiction and potentially the unforeseen collapses in renewable resource stocks (Hepburn, 2003).

3.2 Pessimism about the future

Equation (2) makes it clear that the consumption rate of interest— and thus also the social rate of time preference in a representative agent economy — is a function of consumption growth. If consumption growth, g, will fall in the future, and the utility discount rate, δ, and aversion to fluctuations, μ, are constant, it follows from equation (2) that the social discount rate also declines through time. Furthermore, if decreases in the *level* of consumption are expected — so that consumption growth is negative — the appropriate social rate of time preference could be negative. Declines in the level of consumption are impossible in an optimal growth model in an idealised economy with productive capital. For the social discount rate to be negative, either capital must be unproductive or a distortion, such as an environmental externality, must have driven a wedge between the market return to capital and the consumption rate of interest (Weitzman, 1994).

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[14] Recent work on sin taxes by O'Donoghue and Rabin (2003) provides an example of this type of approach. See also Feldstein (1964), who asks whether the government should act in the best interests of the public, or do what the public wants.

3.3 Intergenerational equity

In an interesting line of research, Chichilnisky (1996, 1997) and Li and Löfgren (2000) create models in which declining discount rates are *necessary* to achieve intergenerational equity (as they define it). Chichilnisky (1996, 1997) requires that the ranking of consumption paths be sensitive to consumption in both the present and the very long run. Li and Löfgren (2000) require that some weight is placed upon the views of both the utilitarians and conservationists in society. While interesting, these theories have not been widely applied, if at all, because the precise mathematical specification of "intergenerational equity" is somewhat arbitrary.   In contrast, considerations of efficiency under uncertainty, addressed next, had led to the adoption of declining discount rate schedules in government policy.

3.4 Uncertainty

It is an understatement to say that we can have little confidence in economic forecasts several decades into the future. In the face of such uncertainty, the most appropriate response is to incorporate it into our economic models. Suppose that the future comprises two equally likely states with social discount rate either 2% or 6%. Discount *factors* corresponding to these two *rates* are shown in Table 1. The average of those discount factors is called the `certainty-equivalent discount factor', and working backwards from this we can find the `certainty-equivalent discount rate', which starts at 4% and declines asymptotically to 2% as time passes.[15] In this uncertain world, a project

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[15] The certainty-equivalent average discount rate is given by $s_c(t) = (1/D_c(t))^{1/t} – 1$, where $D_c(t)$ is the certainty-equivalent discount factor.

is efficient if it passes social cost-benefit analysis using the certainty-equivalent discount rate.

Table 1: Numerical example of a declining certainty-equivalent discount rate

Time (years from present)	1	10	50	100	200	400
Discount factor for 2% rate	0.98	0.82	0.37	0.14	0.02	0.00
Discount factor for 6% rate	0.94	0.56	0.05	0.00	0.00	0.00
Certainty-equivalent discount factor	0.96	0.69	0.21	0.07	0.01	0.00
Certainty-equivalent (average) discount rate	**4.0%**	**3.8%**	**3.1%**	**2.7%**	**2.4%**	**2.2%**

The two key assumptions in this example are that the discount rate is uncertain and *persistent*, so that the expected discount rate in one period is correlated with the discount rate the period before. If these two assumptions hold, intergenerational efficiency requires a declining social discount rate (Weitzman 1998, 2001).

The particular shape of the decline is determined by the specification of uncertainty in the economy. Newell and Pizer (2003) use data on past US interest rates to estimate a reduced-form time series process, which is then employed to forecast future rates. The level of persistence in their forecasts is high enough to generate a relatively rapid decline in the certainty-equivalent discount rate with significant policy implications. However, using the same US interest rate data as Newell and Pizer, Groom et al. (2006) show that misspecification testing suggests employing econometric models which account for second-order dependence and explicitly consider changes in the time series process over time. However, even through the use of such, more flexible and complicated econometric models, the key conclusion remains intact — the certainty-equivalent discount rate declines at a rate that is significant for the appraisal of long-term projects.

Along related lines, Gollier (2001, 2002a, b) analyses an optimal growth model, where a utility function is specified, and demonstrates that a similar result can hold. Specifying a utility function implicitly defines a conception of intergenerational equity, and the optimal solution maximises utility, not the net present value of the cash flows as in Weitzman (1998). Under uncertainty, the social discount rate in equation (2) needs to be modified to account for an additional prudence effect:

$$s = \delta + \mu g - \frac{1}{2} \mu P \operatorname{var}(g) \qquad (3)$$

where P is the measure of relative prudence introduced by Kimball (1990).[16] Prudence measures our propensity to accumulate savings in the face of future risks. This is termed `precautionary saving', and the net impact of the prudence effect is to reduce the discount rate.

These two sets of results show that employing a declining social discount rate is *necessary* for intergenerational efficiency (Weitzman, 1998) and also for intergenerational optimality under relatively plausible utility functions (Gollier 2002a, b). The theory in this section provides a compelling reason for employing declining discount rates in social cost benefit analysis.

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[16] For a conventional utility function *u(c)*, relative prudence is defined as *P = - c u'''(c)/u''(c)*. For a textbook discussion of prudence and precautionary saving, see Romer (2001, Ch 7.6).

3.5 Conclusion

Incorporating uncertainty into social cost benefit analysis leads to the conclusion that a declining social discount rate is necessary for efficient decision-making. Indeed, it was on this basis that the United Kingdom government has incorporated declining social discount rates in its most recent HM Treasury (2003) Green Book, which contains the official guidance on government project and policy appraisal. The arguments based on pessimistic future projections and individual behavioural evidence further support that conclusion. Finally, the fact that declining discount rates also emerge from specifications of intergenerational equity suggests that they help to reduce the tension between considerations of efficiency and intergenerational equity.

3.6 A postscript on time inconsistency

Employing a declining *utility* discount rate can give rise to problems of time inconsistency (Strotz, 1956).[17] Time inconsistency (or 'dynamic inconsistency') arises when a plan, determined to be optimal at one date, is no longer optimal when considered at a later date. In other words, the optimal plan depends upon the evaluation date. In such cases, unless a planner can commit future planners to the original plan, it will eventually be abandoned. Solow (1999) comments that this 'sounds like a poor way to run a railroad.'[18]

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[17] Heal (1998) proves that almost all types of declining utility discount rates, except logarithmic discounting, generate time inconsistency problems.

[18] Hyperbolic discounting has been so successful precisely because this time inconsistency allows it to explain phenomena such as procrastination and addiction, where well-being is not maximised.

Note that the problem of time inconsistency can arise from time-varying *utility* discount rates[19] — it does not arise for time-varying *consumption* discount rates when the underlying *utility* discount rate is constant. As such, declining utility discount rates based upon evidence of individual behaviour, or upon considerations of intergenerational equity, might generate time inconsistency. In contrast, uncertainty in economic growth rates produces declining certainty-equivalent *consumption* discount rates. Provided this is founded upon a constant *utility* discount rate, intertemporal choices will be consistent. They may not always be optimal — in an uncertain world, Newell and Pizer (2003) note that decisions that are sensible *ex ante* may be regrettable *ex post*. But this is not appropriately described as 'time inconsistency'.[20] Additionally, even

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[19] And even with declining *utility* discount rates, one might speculate, given the results in Gollier and Zeckhauser (2005), that there could be a way around the problem. There, although the collective policy shows a declining discount rate, it is not time-inconsistent. This is because each individual agent, with a constant utility discount rate, adopts a time-consistent consumption plan. As such, even though the social planner may be inclined to adjust the plan as time passes, each individual agent has no incentive to do so. So, in theory, decentralisation may effectively create a commitment mechanism that resolves the time inconsistency problem. How this would be implemented in practice is unclear. Perhaps different (but constant) δ could be employed for projects of different lengths?

[20] A problem resembling time inconsistency can emerge in models without explicit utility discount rates, and with declining certainty-equivalent *consumption* discount rates. Weitzman (2001) and Gollier (2004) provide two examples. In such models, Hepburn and Groom (in press) note that the intertemporal conflict arises from time-varying assumptions about discount rate

when time inconsistency is problematic (as it is for declining utility discount rates), it may be of minor concern compared to the frequent policy upheavals caused by changes in the political winds.

4. Forestry case studies

Although declining discount rates have potentially extremely important implications for forestry economics, there does not appear to be much research on the topic. In this section, we seek to remedy that omission by applying a variety of declining discount rate schemes — all motivated by uncertainty in future economic conditions — to three different forestry case studies. We avoid applying discount rates schedules based on other considerations (e.g. intergenerational equity, individual evidence and pessimism about the future) for two related reasons. First, the recent changes in discounting policy relevant to forestry economics have been based on future economic uncertainty. Second, this motivation for declining discount rates is arguable the most robust, as it does not involve controversial (and ultimately political) specifications of intergenerational equity, nor a particular bias in future forecasting, nor the assumption of social irrationality.

The case studies have been selected to illustrate the variety of the impact of using declining discount rates, ranging from virtually no effect to substantial changes in

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uncertainty. In other words, the uncertainty in the discount rate assumed *ex ante* is different to that assumed *ex post*. The problem in such models is not 'time-inconsistency', but rather time-varying assumptions about uncertainty.

profitability estimates. The case studies are hypothetical forestry projects, but are based around real data and advice of forestry economics practitioners.[21]

4.1 Discounting schemes

For each of the case studies, we examine the implications of six discounting schemes. The discounting schemes are chosen to reflect past and current practice in OECD countries, in addition to recent econometric models of the certainty-equivalent discount rate as advanced by Groom et al (2006). The six schemes are:

- *Constant 6% discount rate.* This was guidance in the UK before the recent changes in HM Treasury (2003). It also appears to be close to current guidance of a subset of other OECD countries.

- *Constant 3.5% discount rate.* This is current guidance in the UK for short-term investment projects (under 30 years).

- *Declining 3.5% discount rate.* This follows the step schedule from HM Treasury (2003) shown in Table 2 below. This schedule is based upon a report by OXERA (2002) and the economic theory in Weitzman (1998, 2001) and Gollier (2002).

Table 2: HM Treasury (2003) Green Book Discount Rates

Period of years	0–30	31–75	76–125	126–200	201–300	301+
Discount rate	3.5%	3.0%	2.5%	2.0%	1.5%	1.0%

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[21] We are grateful for the advice of Pat Snowdon at the UK Forestry Commission and Pat Hardcastle of LTS International. Of course, any errors remain our responsibility.

- *AR(4) model.* This model, along with the subsequent two models, attempts to empirically determine the appropriate certainty-equivalent discount rate. Doing this requires some specification of the uncertainty in future discount rates. In this approach, it is assumed that the uncertainty in future discount rates might be similar to the uncertainty in past rates. As such, in the AR(4) model, historical UK interest rate data is employed to estimate an autoregressive process where there are four lags of the dependent variable. On the assumption that the past is informative about the future (which of course is not always correct), this model allows us to specify future discount rate uncertainty, from which a certainty-equivalent discount rate path is derived.

- *Regime-switching model with two states:* This approach is similar to the AR(4) approach, in so far as certainty-equivalent discount rates are derived from past interest rate data. Here, however, we employ a different, more flexible, underlying econometric model. Different econometric models embody different assumptions, so examining a selection of models to determine the preferred approach is advisable. The regime-switching model posits a spot interest rate process that can shift randomly between two regimes. Each regime incorporates a different speed of mean-reversion to a different long-run mean and a different unconditional variance (i.e., the diffusion and drift functions are kept the same but the specific parameter values are different in each regime, leading to a time-heterogeneous process). At any particular point in time there is uncertainty as to which regime applies. The probability of being in each regime at time t is specified as a Markov 1 process, i.e. it depends only on the regime at time t-1.

- *State space model.*[22]   Like the previous two approaches, this approach also determines certainty-equivalent discount rates using past interest rate data. The state space model is an alternative to the regime switching model which is able to capture non-linearities in the mean of the interest rate and to accommodate changes in the conditional variance of the series under consideration. As such, this model captures both the volatility dynamics and the observed non-linearity in the drift of the interest rate process. This time-varying coefficient model can be thought of as an infinite regime-switching model, which allows for a high degree of time-heterogeneity compared with the previous models.

The discount rates that result based upon applying these schemes to UK interest rate data can be determined, and the discount rates for the four time-varying schemes are illustrated in Figure 1 below.

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[22] While the Regime-Switching model allows us to define a finite number of states that the interest rate process goes through, it does not allow for cases where both the level and variance of the process evolve slowly over time. Models with time-dependent parameters can capture such an evolution. Fan et al. (2003) compare various specifications of both time-dependent and time-independent models and propose a time-varying coefficient model, which captures better the time-variation of short-term dynamics of the interest rate. This finding, along with a similar conclusion of Ait-Sahalia (1996), lead Groom et al. (2006) to introduce a time-varying parameter model.
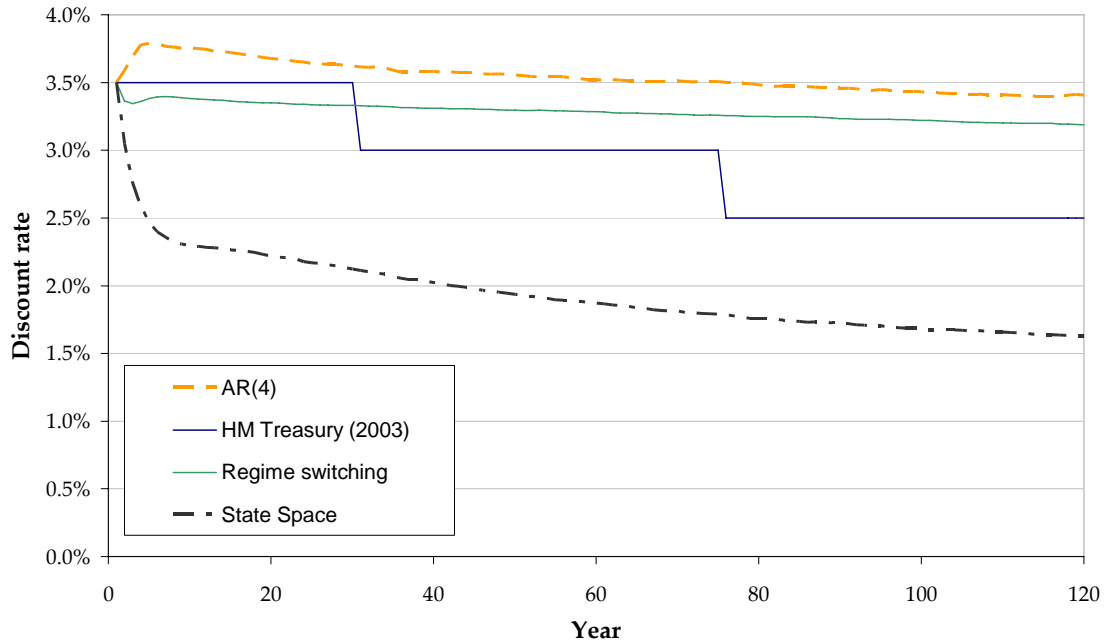
Figure 1: Time-varying discounting schemes

Figure 1 reveals that the path of the certainty-equivalent discount rate differs considerably from one scheme to another. It is clearly important which econometric model is adopted — as indeed we will see in the next section. While Groom et al. (2006) are clear that the regime switching and state space models are to be preferred to the alternatives, deciding between these two latter models is more challenging. A forecasting exercise indicates that the state space model has a lower mean square forecasting error than the regime switching model over a 30-year horizon. As such, they express a tentative preference for the state space model.

The policy implications of model choice are illustrated in the next three sections, where we find net present values for three different forest investments, each with a different time horizon. In particular, we analyse (1) a 22-year harvesting cycle, reflecting a relatively short time horizon which would be applicable for a species such as Ugandan

Pine; (2) a 60-year harvesting cycle appropriate for a medium time horizon; and (3) a 120-year harvesting horizon, applicable for a plantation such as a Scottish Oak.

Of course, the discount rate is itself an important variable determining harvest cycle times. Harvesting should occur once growth rates have slowed to the point where the marginal rate of return equals the discount rate. Lower or declining discount rate schedules will lead to adjustments in thinning times and generally result in longer harvest cycles. Ideally, we would analyse case studies that illustrate the impact of the discount rate on the harvest and thinning path, but for simplicity the case studies below examine fixed harvest cycles.[23]

For clarity, note that the following sections examine projects where the cash flows are *certain* (or at least expressed in *certainty-equivalent terms*), even though other economic conditions are *uncertain* (which is why the efficient discount rate is declining). Uncertainty is relevant to almost all cost-benefit analyses. It is important to distinguish between uncertainty in the project itself and uncertainty about the opportunity cost (the discount rate). For the former, good practice involves quantifying project uncertainty by determining the potential outcomes, assigning appropriate probabilities and applying a coefficient of risk aversion to determine certainty-equivalent flows. Sensitivity analyses can provide useful information. This process (or an alterative) accounts for uncertainty

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[23] Determining these consumption pathways can be rather complex, especially when issues of time inconsistency are relevant, as discussed by Hepburn (2003) in the fisheries context.

in the project. Uncertainty in the opportunity cost is accounted for by using certainty-equivalent discount rates, which tend to be declining, as explained above.

4.2 Short-horizon harvest (22-year cycle)

Our first case study focussed on a forestry investment with relatively short harvesting cycle. The assumptions used in this case study — loosely based upon data from an analysis of *Pinus caribaea* in Uganda — are presented in Table 3.[24] Revenue is derived from thinnings of 33% at ages 8, 12 and 16, but the bulk of the return from the investment arises with complete felling at 22 years.[25] Costs are itemised in Table 3.

Table 3: Assumptions for short-horizon investment

Year(s)	Activity	Real cost/benefit (£/ha)
0	Site preparation	-50
0	Planting	-200
0,12	Road construction	-75
0-60	Annual operating costs	-20
1	Weed control	-50
4,7,10,13	Precommercial thinning/pruning	-15
8	Thinning 1	200
12	Thinning 2	350
16	Thinning 3	500

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[24] The data upon which the assumptions are based was generously provided by Pat Hardcastle, of LTS International. We are also grateful to Pat for his comments and advice on the practical use of discounting in forestry. We alone, however, are responsible for any errors or omissions.

[25] As mentioned above, the thinning and harvesting times are assumed to be fixed, for simplicity. A richer (and more realistic) model would endogenise these variables as a function of the discount rate.

22	Clearfall	4,000

The assumptions in Table 3 were employed to conduct a discounted cash flow analysis for this short-horizon investment. The resulting net present values from the analysis, for each of the six discounting schemes, are presented in Figure 2 below.
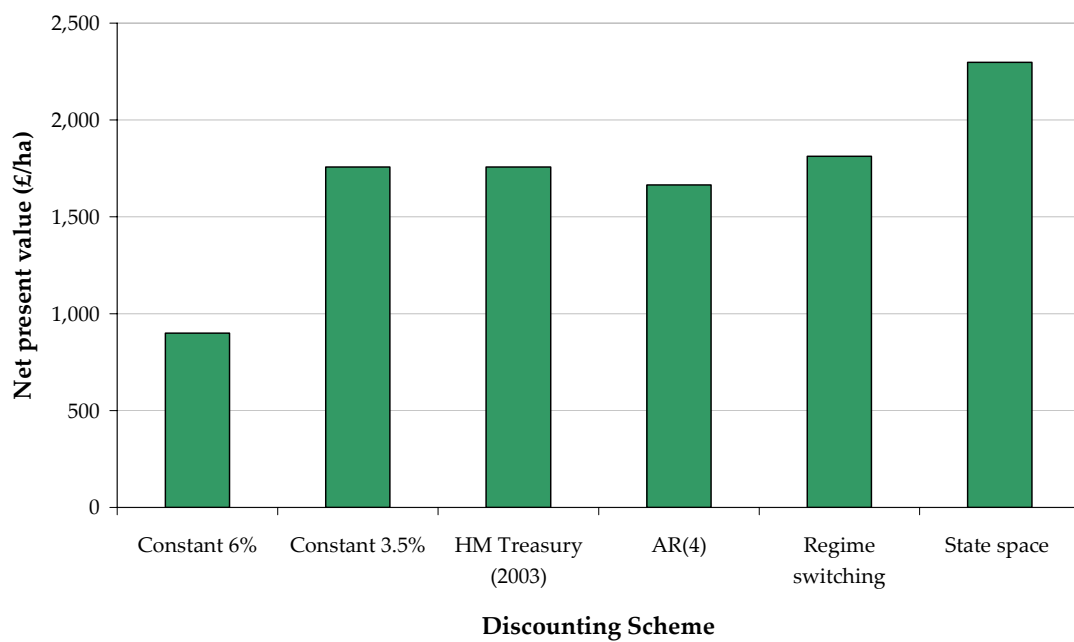
Figure 2: Short-horizon harvest: net present value

Figure 2 shows that, even for the relatively short horizon of 22 years, there is a substantial difference in moving from a constant 6% discount rate to a constant 3.5% discount rate (which is identical to the HM Treasury (2003) result because the discount rate only falls from 3.5% to 3% at year 31). Over this relatively short time horizon, there is very little to be gained in moving to econometric specifications such as the AR(4) or regime switching models. Even using the more sophisticated state space model only increases the net present value by 30% compared with the constant 3.5% discounting scheme.

Overall, the message from this case study is that although the discount rate is an important variable even over relatively short harvest cycles, there is no dramatic difference between a constant 3.5% scheme and the more sophisticated econometric models with declining discount rate. Using a constant discount rate for short time horizons (up to about 30 years) ais likely to be a legitimate approximation.

4.2 Medium-horizon harvest (60-year cycle)

Our medium-horizon case study is based upon the assumptions shown in Table 4.[26] We assume the same initial costs as in the short-horizon case study, but lower annual operating costs. There is only one precommerical thinning, at year 15, and the two commercial thinnings at years 40 and 50 provide a larger contribution to overall revenue than in the previous case study.

Table 4: Assumptions for medium-horizon investment

Year(s)	Activity	Real cost/benefit (£/ha)
0	Site preparation	-50
0	Planting	-200
0,14	Road construction	-75
0-60	Annual operating costs	-10
1	Weed control	-50
15	Precommercial thinning/pruning	-50
40	Thinning 1	1,000
50	Thinning 2	2,000
60	Clearfall	4,000

[26] Figures are based upon hypothetical values from Rose et al (1988). This is a purely hypothetical investment.

Running a discounted cash flow analysis on these assumptions with the six discounting models yields the results in Figure 3.

Figure 3: Medium-horizon harvest: net present value
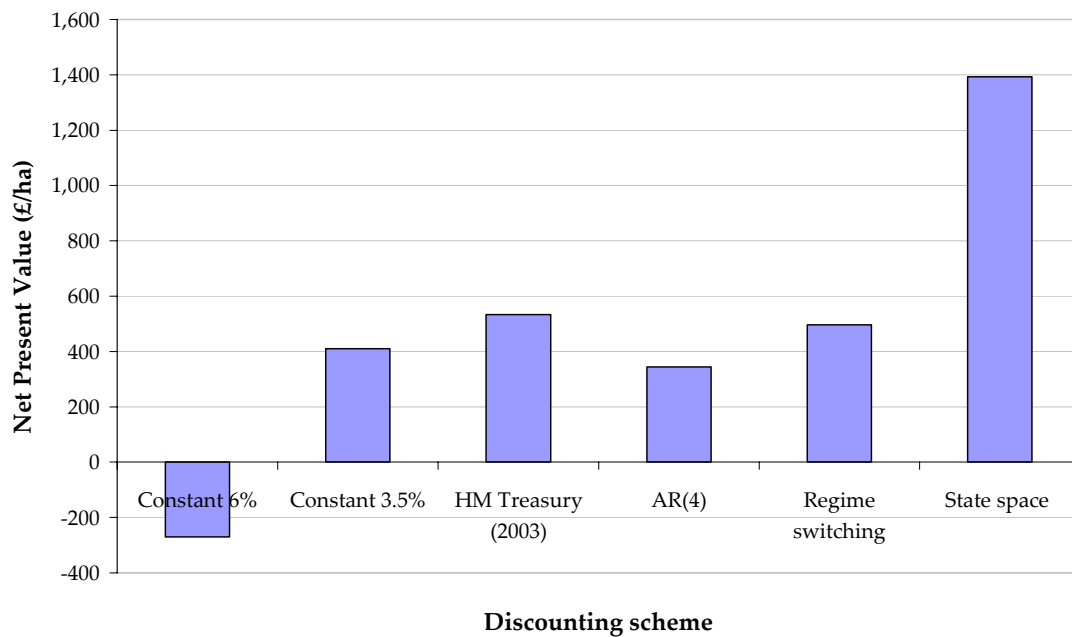
Similar broad conclusions emerge from Figure 3 as in the previous case study. Once again, the shift from a constant 6% to 3.5% discount rate is significant. At a 6% discount rate, the investment does not pass cost-benefit analysis, and we are lead to conclude that the land should be converted to another use. For 3.5% or declining the forestry investment is economically efficient. Unlike the 22-year harvest cycle, here there is a 30% increase when moving from the constant 3.5% discount rate to the HM Treasury (2003) scheme. The AR(4) and regime-switching schemes are less generous to the forestry investment, but this is to be expected given the schedule of discount rates in Figure 1. The most extraordinary result is that the state space scheme increases net

present value by 240% on the constant 3.5% scheme, which is a direct result of the rapid decline in the discount rate under that scheme.

4.3 Long-horizon harvest (120-year cycle)

Our final case study conducts the same discounted cash flows analysis, but for a much longer horizon investment. The assumptions presented in Table 5 are based loosely upon data for investment in a Scottish Oak plantation.[27] We have assumed both higher initial costs and annual administration costs. In contrast to the previous case studies, here clearfall provides less than half of the total (undiscounted) revenue from the plantation.

Table 5: Assumptions for long-horizon investment

Year(s)	Activity	Real cost/benefit (£/ha)
0	Site preparation	-100
0	Planting and tree protection	-800
0-60	Annual administration costs	-20
1	Weed control	-100
5, 20	Precommercial thinning / pruning	-200
60	Thinning 1	4,000
80	Thinning 2	8,000
100	Thinning 3	10,000
120	Clearfall	12,000

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[27] This data was generously provided by Pat Snowdon, of the Forestry Commission Scotland. Again, our assumptions are only loosely based upon the data provided and we remain responsible for any errors or omissions.

Figure 4 presents the results of the discounted cash flow analysis. It is in this case study with such a long time horizon, that the difference between constant and declining discount rates is manifestly apparent. Although the plantation has a negative net present value at a constant 3.5% discount rate, it is profitable under the HM Treasury (2003) declining scheme. In other words, shifting from a constant to a declining discount rate, for the long-horizon harvest analysed here, makes all the difference in deciding whether the scheme is efficient. This underscores the point, made in section 3.5 above, that declining discount rates appear to reduce the tension between efficiency and considerations of intergenerational equity. The variance between the Treasury scheme and the AR(4) and regime switching schemes is larger, again a result of the longer time horizon of this investment. The most striking result, however, emerges from the state space scheme, where the net present value is about £3,000/ha, compared with only £250/ha under the Treasury scheme. The clear conclusion from this final case study is that not only do declining discount rates, the specific form employed is of enormous significance to the outcome of the analysis.

Figure 4: Long-horizon harvest: net present value
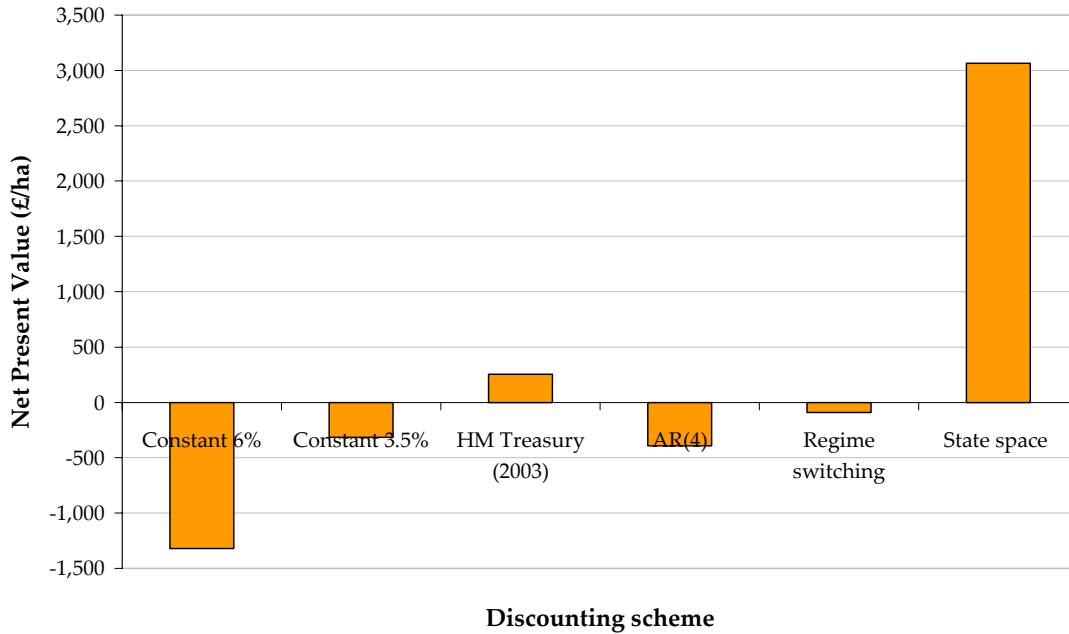
Examining the three case studies overall, we can draw several conclusions. First, the state space and HM Treasury (2003) schemes yield the most favourable results for all three case studies for the simple reason that they involve lower discount rates. Second, the relative results of the different discounting schemes are generally consistent across case studies. Third, the differences between constant and time-varying discounting schemes become more pronounced the longer the time horizon.

5. Conclusions and recommendations

Recent economic theory suggests that appropriate social discount rates should decline with time.. There are several rationales for time-declining rates, but the most important is that the future state of the economy, and thus the appropriate future discount rate, is uncertain. This new body of theory is highly relevant to forestry economics, where time horizons are long and the discount rate plays a central role.

Employing certainty-equivalent discount rates, rather than constant discount rates ignoring uncertainty, is simply intended to ensure that cost-benefit analysis is efficient. In other words, ignoring uncertainty has *inefficiently* biased decision-making towards short-term investments, and away from long-term investments. Using the certainty-equivalent discount rate corrects that bias.

A happy consequence of correcting this bias is a reduction in the tension between intergenerational efficiency and equity. For instance, although shifting from a constant to a declining discount rate made relatively little difference to short-term forestry investments, it had a significant impact on the long-horizon harvest. There, although the investment was inefficient for a constant 3.5% discount rate, it was deemed efficient under the new HM Treasury declining discount rate scheme. In general, shifting to declining discount rate schemes will lead to a shift in the allocation of investment, so that more funds are directed to longer-term projects.

For short-term projects, our case study concords with our intuition that using a constant discount rate will generally be appropriate. However, the choice of the constant discount rate remains significant. For medium-term projects, declining discount rates scheme start to make a difference, and for the investment examined here, although the AR(4) and regime-switching schemes are not particularly generous to the forestry investment, remarkably the state space scheme increases net present value by 240% on the constant 3.5% scheme. For long-term projects, the impact of moving to declining rates is greatest, and again the discounting model matters. The state space scheme

produces the most extreme results, where the net present value is about £3,000/ha, compared with only £250/ha under the HM Treasury scheme.

There are several clear conclusions from our analysis. First, moving to declining discount rates based on uncertainty is theoretically justified. Second, this has important impacts for long-term investments. Third, it has little impact on short-term investments (where a constant discount rate may serve as a legitimate approximation to the correct declining scheme). Fourth, the particular shape of the decline matters a great deal, with substantial differences between the three econometric models examined.

Given these conclusions, it would appear sensible for forestry managers to employ declining discount rates in their economic analysis, which would both increase intertemporal efficiency, and contribute to intergenerational equity and sustainability. Hence our basic recommendation is to build declining discount rates into proprietary software used for forestry financial analysis. Moreover, forest managers will no doubt realise that the implementation of a declining discount rate scheme is not only important for the economic value of future timber products. It is also crucial in determining the net present value of forestry benefits that people derive from forest services, such as extraction of genetic material, tourism, protection of watersheds, support of other ecosystems, carbon storage, etc. Some of these benefits, especially those derived from genetic material and carbon storage, provide very long run benefits, hence are highly sensitive to the choice of the discount rate schedule used in their economic evaluation.

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